Light Work Memo 31: Additive Interferometry Using Horns

Subject: Interferometry
Memo: 31
From: John Makous
Date: 2022 July 11

Additive Interferometry Using Two DSPIRA Radio Horn Telescopes

BACKGROUND

As interferometry is used extensively with radio telescopes, it is of interest to attempt to do interferometry using the DSPIRA radio horn telescopes. In this LightWork memo an investigation of additive interferometry is presented using a 2-horn set-up to observe the transit of the sun. In additive interferometry the voltage signals from the horns are added before the power is calculated by squaring this added input signal. This work was completed by the DSPIRA group over the summers of 2021 and 2022 as part of the DSPIRA RET program. The DSPIRA Lessons webpage has extensive information and lessons on using a horn radio telescope.

ADDITIVE INTERFEROMETRY: BASIC THEORY

The simplest arrangement for radio interferometry consists of 2 horns separated by a baseline distance that is usually parallel to the east-west direction. The horns are pointing in the same direction, as illustrated in Figure 1 below.

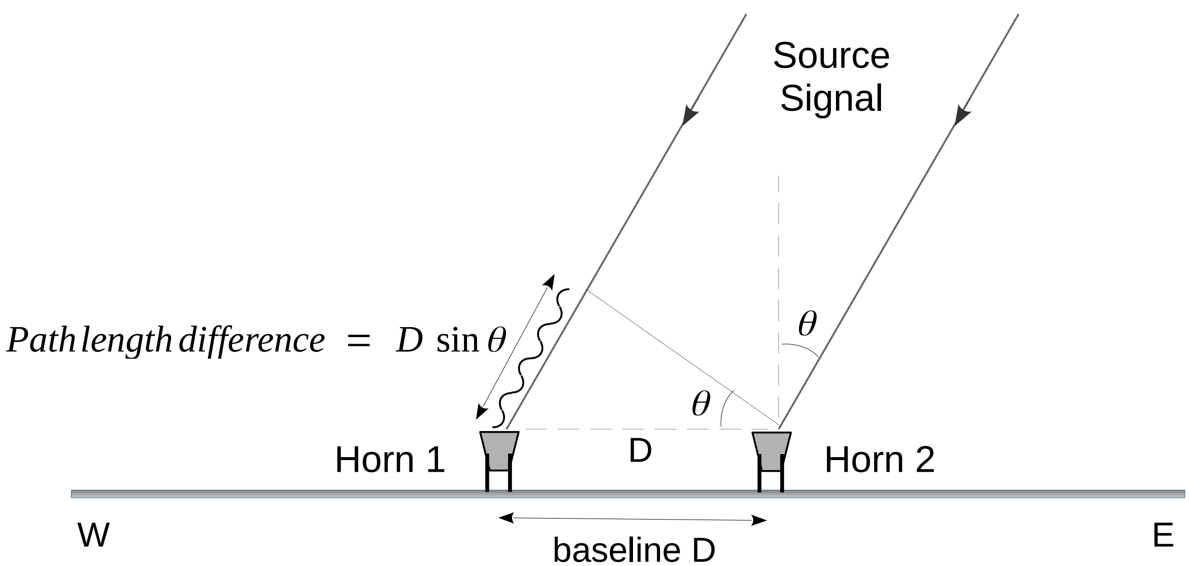

Figure 1. Geometry of a Simple Interferometer

The basic principle of interferometry involves combining the signals received at two different places (i..e the horns) from the same original signal. We will assume that the original source signal is a sinusoidal, monochromatic electromagnetic wave.

The geometry presented in the diagram above illustrates that a given signal from the source will reach Horn 1 later than it reaches Horn 2 due to the extra distance a wave front must travel to reach Horn 1. We will refer to this extra distance as the **path length difference** (PLD). Trigonometry can be used to express the path length difference as

$$PLD = D \sin\theta$$

Similar to the concept behind interference in Young's double slit experiment, constructive interference occurs whenever the path length difference is an integer multiple of the wavelength of the source signal:

$$PLD = m\lambda = D \sin\theta$$ **Condition for Constructive Interference**

where m = 0, 1, 2, ...

The time delay τ_{delay} of the signal reaching Horn 1 after Horn 2 is calculated using the speed of light, c:

$$\tau_{delay} = \frac{PLD}{c} = \frac{D \sin\theta}{c}$$

Using the small angle approximation, $\sin\theta \approx \theta$, this can be expressed as

$$\tau_{delay} \approx \frac{D\ \theta}{c}$$

One difference between radio interferometry and two-slit interference is the range over which the signals can be detected at any instant. In optical interferometry, at a given instant the screen displays a broad range of the interference pattern on a screen. In radio astronomy, the horns represent only one “pixel” on the screen at a given instant. For example, the easiest bright radio source for us to observe with a horn interferometer is the sun, which for our telescopes can be considered a point source. Continuously observing over a span of several hours as the sun passes by is the same as looking at one location on the interference “screen” while moving the source from left to right in front of the slits. So the pattern that is observed in a radio interferometer is a temporal pattern collected at a single point on the screen, not a spatial pattern collected at a given instant.

Figure 2 below illustrates the sequence of combining the signals for additive interferometry.

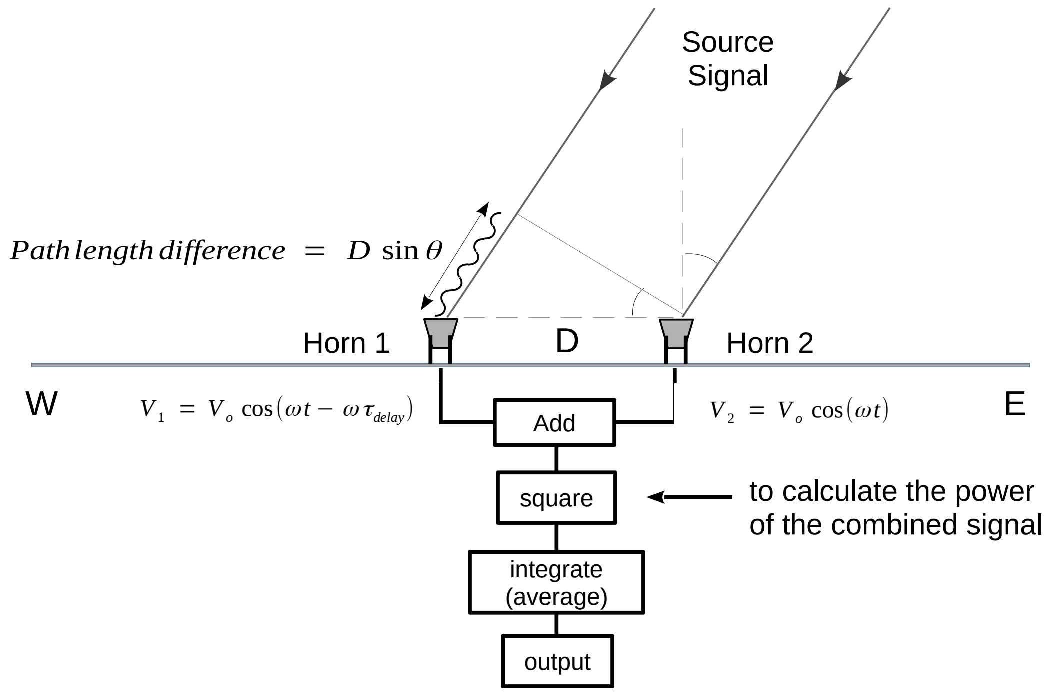

Figure 2. Schematic of An Additive Interferometer

For a sinusoidal, monochromatic signal from the source, the voltages generated at each horn antenna can be written as

$$V_1 = V_o \cos(\omega t - \omega \tau_{delay})$$
$$V_2 = V_o \cos(\omega t)$$

This assumes the amplitude V_o of the signal received at each horn is the same. Their sum is then

$$V_1 + V_2 = V_o \cos(\omega t - \omega \tau_{delay}) + V_o \cos(\omega t)$$

The power of the combined signal received is the square of this sum:

$$\begin{aligned} P &= \int |V_1 + V_2|^2 \, dt = \int [V_1^2 + V_2^2 + 2(V_1 V_2)] \, dt \\ &= \int [{V_o}^2 \cos^2(\omega t - \omega \tau_{delay}) + V_o^2 \cos^2(\omega t) + 2(V_o)^2 \cos(\omega t - \omega \tau_{delay}) \cos(\omega t)] \, dt \\ &= \int [{V_o}^2 \cos^2(\omega t - \omega \tau_{delay}) + V_o^2 \cos^2(\omega t) + (V_o)^2 [\cos(2\omega t - \omega \tau_{delay}) + \cos(\omega \tau_{delay})]] \, dt \end{aligned}$$

constant constant 0

The first two terms average to constant values over time, the third term to zero, and the last term is the interference term that oscillates with a period related to the delay time, τ_{delay}. So

$$P \propto \cos \omega \tau_{delay} ,$$

and the fringe pattern should repeat at a change in angle $\Delta\theta$ according to the following:

$$\omega\,\tau_{delay} = 2\pi$$

$$2\pi f\left(\frac{D\,\Delta\theta}{c}\right) = 2\pi f\left(\frac{D\,\Delta\theta}{(f\lambda)}\right) = 2\pi \rightarrow \Delta\theta = \frac{\lambda}{D}$$

NOTE: This is a simplified presentation of what actually occurs. An accurate analysis requires adding a secondary term involving $\sin(\omega t)$, and the signals are more conveniently expressed using complex numbers. The simplification presented here is to illustrate the principle of interference that might be used with students.

EXPERIMENTAL SET UP

Two horns are set up along the east-west direction separated by the desired distance. A Lime software defined radio (SDR) is capable of processing 2 inputs; so it is used as the SDR for interference. We use the program *interferometer_Lime_simpleSpectrometer_adding.grc* as the spectrometer.

A photograph of the set-up we used (actually done at the Green Bank Observatory) is shown in Figure 3.

Details on the experimental set-up can be found in the document titled "*Setting Up 2 Horn Interferometer*" on the DSPIRA Lessons website (https://wvurail.org/dspira-lessons/tour/).

Figure 3. DSPIRA 2-Horn Interferometer

The flowchart for this spectrometer is shown in Figure 4 below. The parts of the flowchart that correspond to the schematic diagram in Figure 2 are highlighted in red for easy identification. This flowchart also saves the spectra from the individual telescopes (outlined in green) for post analysis by the user if desired.

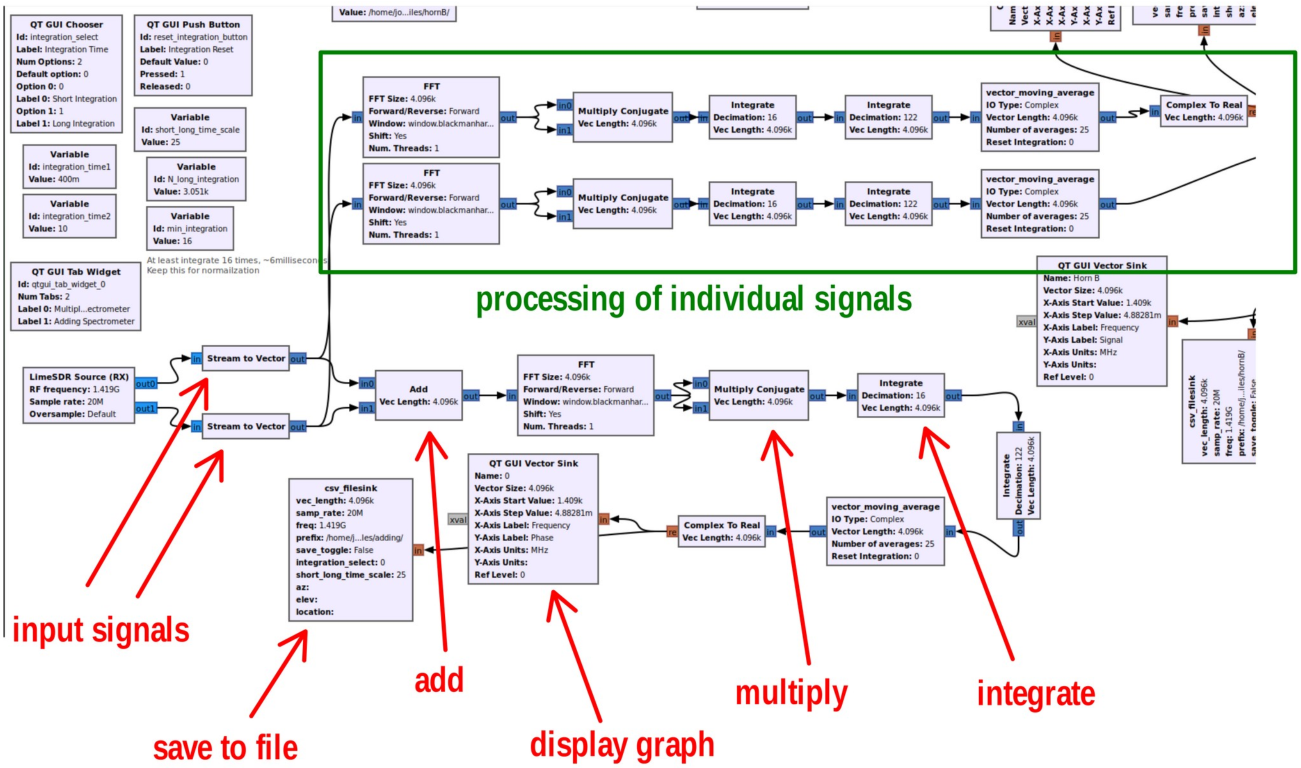

Figure 4. *GNURadio* Spectrometer for Additive Interferometer

DATA ANALYSIS

The *interferometer_Lime_simpleSpectrometer_adding.grc* spectrometer is designed to collect spectra once every integration time. The integration time is selected to be long enough for good averaging, e.g. 5 seconds, but not too long so as to not have a good resolution of the interference fringes.

To generate a graph of output signal vs. incident angle θ, it is necessary to calculate the area of each spectrum collected at each integration time. A scan with a 5 s integration time corresponds to 720 spectrum files collected per hour. For a sun a scan may last approximately 4 hours, which results in lots of data files. A Python program was used to calculate the areas under each spectrum using a trapezoid method. The spectrum range was limited to 1416 MHz to 1424 MHz to keep the range close to the optimal frequency of 1420.4 MHz that the horn telescopes are designed for. Since each spectrum file collected has a time stamp, the sun's angle relative to transit can be determined. As a result, we are able to graph the Interference Magnitude vs. Angle.

The graph in Figure 5 below shows the output of a 2-horn additive interferometer used to observe a transit of the sun centered about its transit. The baseline distance was 5.0 m, and an integration time of 5 s was used. The angle on the *x*-axis is from the start of the scan, not the transit time.

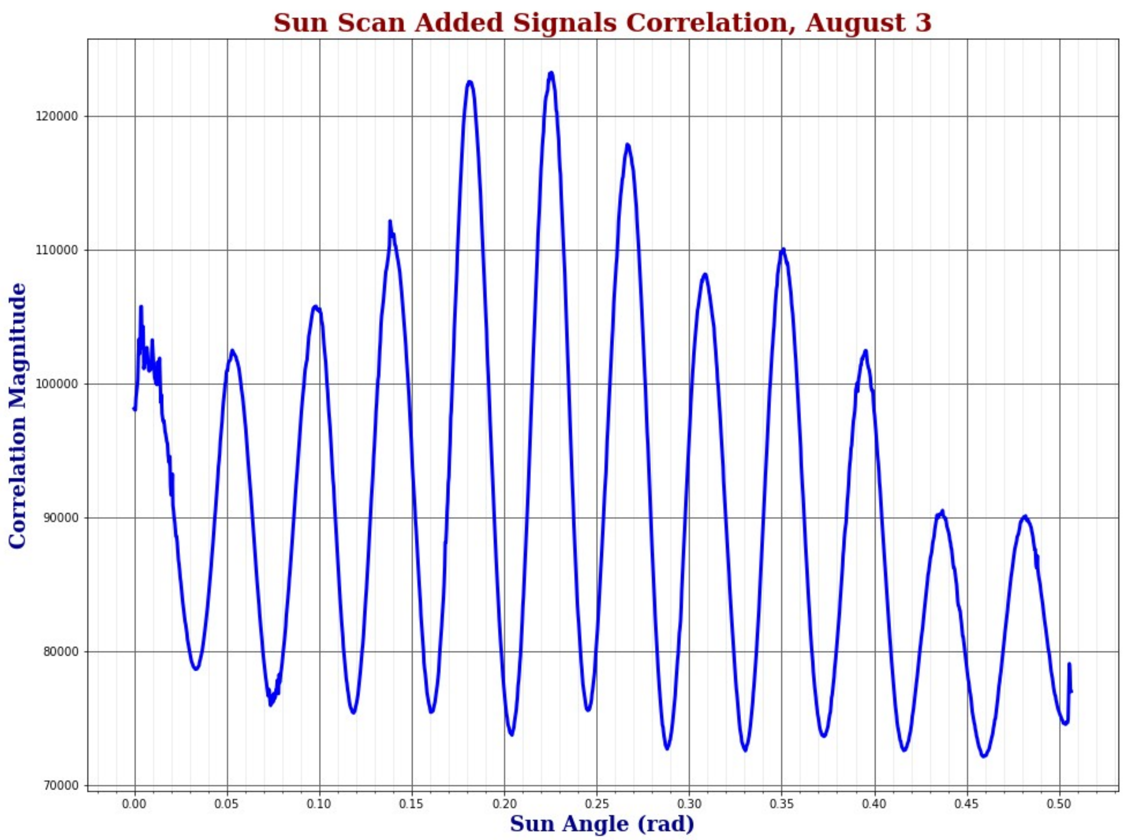

Figure 5. Additive interference pattern of the sun with D = 5.0 m.

According to the theory the pattern should repeat when

$$\Delta\theta = \frac{\lambda}{D} = \frac{0.21\ m}{5.0\ m} = 0.042\ rad$$

The graph in Figure 4 shows an angular spacing of 0.043 rad, in excellent agreement.

FUTURE WORK

Another type of interferometry involves multiplying the signals from the horns rather than adding them. We have successfully done this using the horns and seen interference phenomena, but the analysis is a little more complicated. This work will be presented in a future LightWork memo.

Other projects to be investigated include trying to observe other objects using interferometry, such as the moon or Cassiopeia A, and also trying to do standard beamforming.